

S-brane to thermal non-singular string cosmology

Costas Kounnas¹, Hervé Partouche² and Nicolaos Toumbas³

¹ Laboratoire de Physique Théorique, Ecole Normale Supérieure,[†]
24 rue Lhomond, F-75231 Paris cedex 05, France
Costas.Kounnas@lpt.ens.fr

² Centre de Physique Théorique, Ecole Polytechnique,[‡]
F-91128 Palaiseau cedex, France
herve.partouche@cpht.polytechnique.fr

³ Department of Physics, University of Cyprus,
Nicosia 1678, Cyprus.
nick@ucy.ac.cy

Abstract

We present a new class of non-singular string cosmologies in d space-time dimensions. At very early times, $\tau \ll \tau_c$, the Universe is described by a flat σ -model metric, a constant maximal temperature T_c and super-weak string interactions, $g_{\text{str}} \ll 1$. During the evolution, the metric remains flat up to τ_c , while the string coupling grows and reaches a critical value g_{str}^* at τ_c . This phase is characterized by a uniform temporal distribution of spacelike branes. At later times, $\tau > \tau_c$, the Universe enters in a new phase of expansion, with radiation. The string coupling decreases due to the dilaton motion and asymptotes to a constant for $\tau \gg \tau_c$. Throughout the evolution, the string coupling remains smaller than g_{str}^* . In the Einstein frame, the cosmologies describe bouncing Universes, where two distinct phases are connected at τ_c . In the initial contracting phase, the evolution of the scale factor is identical to that of a negatively curved Universe filled with radiation. At later times, the Universe enters in an expanding thermal phase with a running dilaton. Explicit examples are presented in a large class of thermal $(4, 0)$ type II superstring vacua, with non-trivial “gravito-magnetic” fluxes.

[†] Unité mixte du CNRS et de l'Ecole Normale Supérieure associée à l'Université Pierre et Marie Curie (Paris 6), UMR 8549.

[‡] Unité mixte du CNRS et de l'Ecole Polytechnique, UMR 7644.

1 Introduction

In string theory, one expects a drastically different cosmological picture to emerge, as compared to the conventional field theory scenario [1], especially at very early cosmological times, where new stringy degrees of freedom become relevant, dominating the high temperature and high curvature regimes [2–4]. In these extreme situations, purely stringy phenomena occur, which do not admit conventional field theory descriptions [5]. String oscillators and winding states become relevant around the Hagedorn temperature T_H , before the onset of curvature singularities, and drive a phase transition towards a non-trivial thermal vacuum [2, 3, 6–13].

Non-singular cosmological solutions in various space-time dimensions were recently found, based on a mechanism which resolves the Hagedorn instabilities of strings at finite temperature [14, 15]. The key ingredients of this mechanism were shown to be generic in a large class of initially $\mathcal{N}_4 = (4, 0)$ superstring models, where finite temperature is introduced along with special “*gravito-magnetic fluxes*”. The fluxes modify the thermal vacuum by injecting into it non-trivial momentum and winding charges, lifting the Hagedorn instabilities of the canonical ensemble [14–18]. The fundamental property of the new vacuum is the restoration of thermal T-duality symmetry, implying a maximal critical temperature T_c , which occurs at the self-dual point. The duality invariant temperature can be written as $T = T_c e^{-|\sigma|}$, where σ is the thermal modulus parametrizing the radius of the Euclidean time circle, $R_0 = R_c e^\sigma$. It turns out that the right-moving sector gives rise to an *asymptotically supersymmetric structure* [19, 20] that restores thermal T-duality symmetry, avoiding at the same time the Hagedorn tachyonic instabilities [15].

In all such thermal stringy systems, there are three characteristic regimes, each with a distinct effective field theory description: Two dual asymptotically cold regimes associated with the light thermal momentum and light thermal winding states respectively, and the intermediate regime where additional massless thermal states appear, leading to enhanced Euclidean gauge symmetry. The extra massless states source spacelike branes, localized at the critical points, which glue together the Momentum and Winding regimes. Thus the intermediate phase comprises a “Brane regime”. As shown in [14, 15], the thermal partition function exhibits a conical structure as a function of the thermal modulus σ , irrespectively of the dimensionality of the model. Thanks to the *asymptotically right-moving supersymmetric structure*, in both the Momentum $\{\mathcal{M}(\sigma > 0)\}$ and Winding $\{\mathcal{W}(\sigma < 0)\}$ regimes, the

energy density and pressure can be well-approximated by the energy density and pressure of massless thermal radiation up to the critical temperature [15]:

$$\rho \simeq (d-1)P, \quad P \simeq n^* \Sigma_d T^d, \quad T = T_c e^{-|\sigma|}. \quad (1.1)$$

Here, Σ_d is the Stefan-Boltzmann constant and n^* is the number of initially massless states, modulo a spin-statistics factor for fermions:

$$\Sigma_d = \frac{\Gamma(d/2)}{\pi^{d/2}} \zeta(d), \quad n^* = n^B + n^F \left(\frac{2^{d-1} - 1}{2^{d-1}} \right).$$

The conical singularity in σ is resolved by *the spacelike branes*, which comprise the “Brane regime” $\{\mathcal{B}(\sigma = 0)\}$ at the critical point $\sigma = 0$. These branes provide localized (in time) negative pressure contributions, which turn out to be crucial in evading the constraints imposed by the singularity theorems of classical general relativity [1] on realizing singularity-free, bouncing cosmologies.

Utilizing the ingredients above, a string effective Lorentzian action covering simultaneously the three characteristic regimes was obtained in Refs [14,15]. This action incorporates the spacelike branes that glue together:

- i) the *Winding regime* $\equiv \{\mathcal{W}(\sigma < 0; \tau < \tau_c)\}$,
- ii) the *Brane regime* $\equiv \{\mathcal{B}(\sigma = 0; \tau = \tau_c)\}$ and
- iii) the *Momentum regime* $\equiv \{\mathcal{M}(\sigma > 0; \tau > \tau_c)\}$

at a given time τ_c . The non-singular string cosmology can be viewed as the gluing at τ_c of the above three mentioned regimes:

$$\{\mathcal{C}_{\text{String}}(\tau)\} \equiv \{\mathcal{W}(\tau < \tau_c)\} \oplus \{\mathcal{B}(\tau = \tau_c)\} \oplus \{\mathcal{M}(\tau > \tau_c)\}. \quad (1.2)$$

We would like to stress here that the existence of string mechanisms gluing distinct effective field theories were conjectured in the past by several authors and in several related contexts, like for instance the gluing of the $\mathcal{N} = 2$ Calabi-Yau theories with the $\mathcal{N} = 2$ Landau-Ginsbourg theories [21], or even the gluing of theories related by S and/or T-dualities [5,22]. In the majority of examples in the literature, the precise gluing mechanism could not be well established, since the intermediate region corresponded to a non-perturbative regime, and therefore the description was technically uncontrollable.

The aim of this work is to investigate the possibility of constructing new interesting d -dimensional string cosmologies, $\{\tilde{\mathcal{C}}_{\text{String}}(\tau)\}$, obtained by gluing at τ_c a Brane regime

$\{\mathcal{B}(\tau \leq \tau_c)\}$, which lasts arbitrarily long in time, with the Momentum $\{\mathcal{M}(\tau > \tau_c)\}$ regime:

$$\{\tilde{\mathcal{C}}_{String}(\tau)\} \equiv \{\mathcal{B}(\tau \leq \tau_c)\} \oplus \{\mathcal{M}(\tau > \tau_c)\}. \quad (1.3)$$

Thus, initially the Universe is described by the Brane regime. During this phase, the σ -model temperature and scale factor remain constant, with the temperature maintained at its maximal critical value T_c , while the string coupling grows from super-weak values in the very early past, reaching a maximal value g_{str}^* at τ_c . Just after this moment, the Universe exits into the radiation dominated Momentum regime. In the Einstein frame, the whole evolution describes a bouncing Universe, with two distinct phases connected at $\tau = \tau_c$. In the initial contracting phase associated to the Brane regime, the scale factor evolves as in a negatively curved Universe filled with radiation. In the Momentum regime, the Universe is expanding with thermal radiation and a running dilaton. As we will show, the Brane regime can be understood in terms of a temporal distribution of the thin spacelike branes which appear in the cosmologies of [14,15] at the critical point $\sigma = 0$. The microscopic origin of the effective action in the Brane regime follows from the underlying stringy description of the thermal system at the extended symmetry point $\sigma = 0$, incorporating various fluxes in the effective gauged supergravity theory. The whole of the cosmological evolution can be treated perturbatively provided that the critical value of the string coupling g_{str}^* is small enough.

The plan of the paper is as follows. In Section 2, we briefly review some of the type II thermal vacua, which are free of Hagedorn instabilities due to the presence of non-trivial gravito-magnetic fluxes. Some of their characteristic properties are displayed, such as the universal conical structure of the thermal partition function, the existence of a maximal critical temperature T_c , as well as the existence of the three distinct effective field theory descriptions associated with the Winding regime $\{\mathcal{W}(\sigma < 0)\}$, the Brane regime $\{\mathcal{B}(\sigma = 0)\}$ and the Momentum regime $\{\mathcal{M}(\sigma > 0)\}$. In Section 3, we present the effective actions valid in each of the three phases respectively. Special attention is devoted to the Brane regime. The origin of the latter is purely stringy, characterized by an enhancement of the Euclidean-time $U(1)_L$ gauge symmetry to an $[SU(2)_L]_{k=2}$ symmetry at T_c . At this critical temperature, additional massless thermal states source localized negative pressure contributions to the effective action, admitting a spacelike brane interpretation. We explore the possibility of distributing the branes in the time interval $\tau \leq \tau_c$. The consistency of such a distribution as well as the constraints imposed on the general structure of the Brane effective potential

are analyzed in Section 3.1. The distribution of branes gives rise to an interesting cosmology with the Universe being initially in the Brane regime $\{\mathcal{B}(\tau \leq \tau_c)\}$ up to τ_c . This early times cosmological phase, as well as the Winding and Momentum cosmological phases are described in Sections 3.2 and 3.3. In Section 3.4, the gluing of the initial Brane regime $\{\mathcal{B}(\tau \leq \tau_c)\}$ with the Momentum regime $\{\mathcal{M}(\tau > \tau_c)\}$ is presented. The resulting string cosmologies $\{\tilde{\mathcal{C}}_{\text{String}}(\tau)\} \equiv \{\mathcal{B}(\tau \leq \tau_c)\} \oplus \{\mathcal{M}(\tau > \tau_c)\}$, described in Section 3.5, are free of initial curvature singularities and are compatible with string perturbation theory throughout the evolution. In Section 4, we present the microscopic origin of the Brane effective action, and in particular the Brane effective potential, in terms of non-trivial fluxes, making use of the underlying gauged supergravity effective description of the stringy extended symmetry point. Finally, our results and perspectives are summarized in Section 5.

2 Modified thermal ensemble with fluxes, temperature duality and the Hagedorn transition

In this Section, we review the key properties of tachyon-free thermal configurations associated to type II $\mathcal{N}_4 = (4, 0)$ vacua in various dimensions. At zero temperature, the left-moving worldsheet degrees of freedom give rise to 16 real supercharges, while the remaining right-moving supersymmetries are broken spontaneously via asymmetric geometrical fluxes [14–18]. At special values of the moduli participating in the breaking of the right-moving supersymmetries, the local gauge symmetry is enhanced to a non-Abelian gauge symmetry. Thermal and quantum effects can stabilize the system at such extended symmetry points [23–25].

Finite temperature is introduced along with non-trivial gravito-magnetic fluxes, threading the Euclidean time cycle together with other cycles responsible for the breaking of the right-moving supersymmetries [14–18]. These fluxes inject into the thermal vacuum non-trivial momentum and winding charges and lift the Hagedorn instabilities of the canonical ensemble, leading to a restoration of thermal duality symmetry for the partition function: $Z(\beta/\beta_c) = Z(\beta_c/\beta)$. Here β denotes the period of the Euclidean time cycle, attaining a critical value β_c at the self-dual point. In all these models the absence of the conventional exponential growth of thermally excited massive states is due to the presence of asymptotic

supersymmetry coming from the right-moving sector. Although our explicit examples of tachyon-free thermal superstring models, which are described in great detail in the literature, involve type II initially $(4, 0)$ vacua, we believe that the key properties, as identified in these models, will be generic in all thermal, tachyon-free superstring realizations¹.

The main properties which lead to the resolution of the Hagedorn and initial singularities are exhibited below:

- The gravito-magnetic fluxes render the spectrum of thermal masses semi-positive definite. Consequently, the partition function is finite for all values of β and duality invariant under $\beta \rightarrow \beta_c^2/\beta$. At the critical point $\beta = \beta_c$, new massless thermal states appear, extending the Euclidean-time $U(1)_L$ gauge symmetry to a non-Abelian $[SU(2)_L]_{k=2}$ symmetry. This phenomenon is absent in any conventional field theory model. In all string models, the self-dual point $\beta = \beta_c$ occurs at the fermionic point.
- The extra massless states at the critical point possess non-trivial momentum and winding charges along the Euclidean-time circle, so that $p_L = \pm 1$ and $p_R = 0$. These two extra states together with the thermal radius modulus give rise to the $SU(2)$ enhanced symmetry. At the critical point, the massless states give rise to non-trivial backgrounds, which admit a localized brane interpretation (in the Euclidean).
- For $\beta/\beta_c \gg 1$, the thermal partition function is dominated by the light thermal momentum states, giving rise to the characteristic behavior of massless thermal radiation in d dimensions, modulo exponentially suppressed contributions from the massive string oscillator states:

$$\frac{Z}{V_{d-1}} = \frac{n^* \Sigma_d}{\beta_c^{d-1}} \left(\frac{\beta_c}{\beta} \right)^{d-1} + \mathcal{O}(e^{-\beta/\beta_c}), \quad (2.1)$$

where n^* counts the number of effectively massless degrees of freedom, Σ_d is the Stefan-Boltzmann constant for radiation and V_{d-1} is the spatial volume. Thanks to the thermal duality symmetry, the asymptotic behavior for $\beta/\beta_c \ll 1$ is dual-to-thermal, dominated by the light thermal winding states:

$$\frac{Z}{V_{d-1}} = \frac{n^* \Sigma_d}{\beta_c^{d-1}} \left(\frac{\beta}{\beta_c} \right)^{d-1} + \mathcal{O}(e^{-\beta_c/\beta}). \quad (2.2)$$

¹Similar behavior is exhibited in the two-dimensional heterotic strings [10].

Here also, the oscillator states give exponentially suppressed contributions. The contribution of the massive oscillator states remains finite at the critical point, as the fluxes effectively reduce the density of thermally excited massive states. In most cases, the contribution of the massive oscillator states never dominates over the thermally excited massless states, due to asymptotic supersymmetry coming from the right-moving sector [14–20].

- *The duality invariant temperature T* , valid in both asymptotic thermal regimes, is given by $T \equiv T_c e^{-|\sigma|}$, where we have defined the thermal modulus σ by $e^\sigma = \beta/\beta_c$. As a result, the temperature, the energy density and pressure in these configurations never exceed certain maximal values. The maximal critical temperature is given by $T_c = 1/\beta_c$. In both asymptotic regimes ($T \ll T_c$), the partition function can be expressed in terms of the self-dual temperature as follows:

$$\frac{Z}{V_{d-1}} = n^* \Sigma_d T_c^{d-1} \left(\frac{T}{T_c} \right)^{d-1} + \mathcal{O}(e^{-T_c/T}). \quad (2.3)$$

- Due to the right-moving asymptotic supersymmetry, the behavior of the thermal partition function $Z(\sigma)$ is controlled by the thermally excited massless states everywhere and up to the critical point $\sigma = 0$ [15]:

$$\frac{Z}{V_{d-1}} \simeq n^* \Sigma_d T_c^{d-1} e^{-(d-1)|\sigma|}, \quad (2.4)$$

modulo the exponentially suppressed contributions in all aforementioned regimes. This result reveals a universal conical structure as a function of the thermal modulus σ , irrespectively of the dimensionality of the model. The conical structure at the critical point is naturally resolved by the presence of additional massless thermal states. This result also implies that in each of the two thermal phases corresponding to the light thermal momenta and light windings, the various thermodynamical quantities enjoy the standard monotonicity properties as functions of the temperature, with the specific heat being positive up to the critical point.

- The presence of the localized massless states is crucial since *they can marginally induce transitions between purely momentum and purely winding states*, thus driving a phase transition between the two dual asymptotically cold regimes. As in [14, 15], this phase transition admits an almost geometrical description, in terms of a “T-fold” [26, 27],

with branes localized at the critical point gluing the “Momentum” and “Winding” spaces. The branes provide localized (in time) negative pressure contributions to the effective action.

We conclude that the stringy thermal system has three characteristic regimes: The regime of light thermal windings, $\{\mathcal{W}(\sigma < 0)\}$, the dual regime of light thermal momenta, $\{\mathcal{M}(\sigma > 0)\}$, and the third intermediate brane regime, $\{\mathcal{B}(\sigma = 0)\}$, corresponding to the extended symmetry point, where additional massless thermal states carrying non-trivial momentum and winding charges become relevant. In Refs [14, 15], it was shown how to realize non-singular string cosmologies via a stringy gluing mechanism of the three regimes, which incorporates spacelike branes at a certain time τ_c when the temperature reaches its critical value:

$$\{\mathcal{C}_{\text{String}}(\tau)\} \equiv \{\mathcal{W}(\tau < \tau_c)\} \oplus \{\mathcal{B}(\tau = \tau_c)\} \oplus \{\mathcal{M}(\tau > \tau_c)\}. \quad (2.5)$$

The ingredients described above not only treat successfully the Hagedorn transition, but lead also to non-singular thermal cosmologies in contrast to field theoretic cases.

The advantage of the above described cosmological examples is that the stringy gluing mechanism turns out to be under perturbative control, for all cosmological times, both under the string coupling g_{str} and the α' -expansions. Indeed, the Brane regime turns out to be described by an exact conformal field theory, which can be treated beyond any α' approximation. Furthermore, the string coupling turns out to be bounded by a critical coupling $g_{\text{str}}^* = g_{\text{str}}(\tau_c)$, which can be chosen to be sufficiently small so that both the localized negative pressure contribution from the Brane regime as well as the bulk thermal corrections arising from the Winding and Momentum regimes can be determined unambiguously. These facts allow us to realize the gluing stringy mechanism and to obtain the non-singular cosmological evolution $\{\mathcal{C}_{\text{String}}(\tau)\}$ valid at all times. The resulting non-singular string cosmologies $\{\mathcal{C}_{\text{String}}(\tau)\}$ describe bouncing Universes, which in the asymptotic regimes $|\tau| \gg 1$ can be either radiation or curvature dominated [15]. The bounce occurs at the brane regime $\{\mathcal{B}(\tau_c)\}$, where a phase transition occurs between the Winding $\{\mathcal{W}(\tau < \tau_c)\}$ and the Momentum $\{\mathcal{M}(\tau > \tau_c)\}$ regimes. These bouncing cosmologies are the first higher dimensional examples, where both the Hagedorn singularity and the classical Big Bang singularity are successfully resolved, remaining perturbative throughout the evolution.

In this work we explore a new class of d -dimensional cosmologies, $\{\tilde{\mathcal{C}}_{\text{String}}(\tau)\}$, obtained

by gluing at τ_c a Brane regime $\{B(\tau \leq \tau_c)\}$ with the Momentum regime $\{\mathcal{M}(\tau > \tau_c)\}$:

$$\{\tilde{\mathcal{C}}_{\text{String}}(t)\} \equiv \{\mathcal{B}(\tau \leq \tau_c)\} \oplus \{\mathcal{M}(\tau > \tau_c)\}. \quad (2.6)$$

We would like to examine under what circumstances the Universe will remain in the Brane regime at the early times $\tau \leq \tau_c$, while for $\tau > \tau_c$ will enter in the Momentum regime described by thermal radiation.

3 The three effective field theory actions up to genus-1

The above discussion and the results of Refs [14, 15], summarized in the previous Section, clearly show that the cosmological evolution depends crucially on the value of the thermal modulus σ . There are three possible effective field theory actions associated to distinct cosmological time intervals: *i*) the Winding-action \mathcal{S}_W , which is restricted to $\sigma(\tau) < 0$, *ii*) the Brane-action \mathcal{S}_B , with $\sigma(\tau) = 0$ and *iii*) the Momentum-action \mathcal{S}_M with $\sigma(\tau) > 0$.

During the cosmological evolution, the thermal modulus σ acquires non-trivial time-dependence, $\sigma(\tau)$, and so the Universe may pass through all of the three regimes. The transition from the Winding to the Momentum regime (and vice versa), necessarily crosses the Brane regime, where the Universe can stay in principle for a certain amount of time. Therefore, in order to study the stringy cosmological evolution, it is necessary not only to derive the three effective actions, but also to specify the gluing mechanism of the Winding with the Brane regime as well as the Brane with the Momentum regime.

The Momentum and Winding regimes can be easily described by effective d -dimensional dilaton-gravity σ -model actions (up to the genus-1 level):

$$\begin{aligned} \mathcal{S}_M &= \int d^d x \sqrt{-g} \Theta(\sigma) \left\{ e^{-2\phi} \left[\frac{\mathcal{R}}{2} + 2(\partial\phi)^2 + \dots \right] + P(\sigma) \right\} \\ \mathcal{S}_W &= \int d^d x \sqrt{-g} \Theta(-\sigma) \left\{ e^{-2\phi} \left[\frac{\mathcal{R}}{2} + 2(\partial\phi)^2 + \dots \right] + P(-\sigma) \right\}. \end{aligned} \quad (3.1)$$

The term proportional to the pressure P in both expressions is the genus-1 contribution of the thermal effective potential, while the rest comprise the genus-0 effective dilaton-gravity action written in the σ -model frame. The ellipses denote the contributions of the two-index antisymmetric tensor, other moduli fields, gauge bosons and space-time fermions. The two

actions, modulo the Θ -constraints, look identical. This is a consequence of thermal duality, which implies that the pressure

$$P \simeq n^* \Sigma_d T_c^d e^{-d|\sigma|} \quad (3.2)$$

is a duality invariant quantity, even though the spinor representations in the Winding regime have opposite chirality from those in the Momentum regime [14–16]. The Spinor and anti-Spinor contributions to the thermal effective potential amount to identical results. Moreover, all thermodynamical quantities such as the temperature, the energy density and the pressure are given in terms of manifestly duality invariant expressions involving the absolute value of the thermal modulus $|\sigma|$. Thus thanks to thermal duality, both the Momentum and Winding regimes can be simultaneously described by *a unique expansion in terms of the duality invariant temperature* $T = T_c e^{-|\sigma|}$.

The Brane regime appears when $\sigma = 0$. Its origin is purely stringy, characterized in the Euclidean by extra massless thermal states carrying non-trivial winding and momentum charges. The existence of such states is crucial for realizing the gluing mechanism between the Winding, Brane and Momentum phases. It is also interesting that the Brane regime is well described in terms of an $[SU(2)_L]_{k=2}$ conformal field theory associated with the fermionic extended symmetry point. In this regime, we have to include the genus-0 backgrounds of the extra massless thermal states. These tree level contributions admit a brane interpretation [14, 15]. The brane tension is determined by the allowed non-trivial backgrounds of the extra massless thermal scalars $\langle \partial\varphi^I \rangle^2 \neq 0$, where there are non-trivial gradients along the directions transverse to Euclidean time, as well as non-trivial fluxes $\langle F_{IJ}^a \rangle \neq 0$ associated to the extra gauge bosons appearing at τ_i with $\sigma(\tau_i) = 0$. The non-zero values of $\langle \partial\varphi^I \rangle$ and $\langle F_{IJ}^a \rangle$ give rise to an effective localized Brane-action with a non-trivial potential for the dilaton field:

$$\mathcal{S}_B(\tau_i) = \int d\tau \delta(\tau - \tau_i) d^{d-1}x \sqrt{g_\perp} \left\{ e^{-2\phi} \left[\frac{\mathcal{R}}{2} + 2(\partial\phi)^2 \right] + P - V_i(\phi) \right\}. \quad (3.3)$$

On shell, the thermal modulus is identically vanishing and the genus-1 contribution to the pressure is $P_c := P(\sigma = 0)$. The microscopic origin of the above action and the precise structure of $V_i(\phi)$ will be presented in Section 4. In what follows we treat $V_i(\phi)$ as a generic function of ϕ , specified when necessary.

In Refs [14, 15] the possibility that the brane appears only once at τ_c (with $\sigma(\tau_c) = 0$) was

investigated. The resulting cosmology gives rise to a bouncing Universe gluing together at τ_c a “Winding” contracting phase with a “Momentum” expanding phase. However, the brane picture suggests for another interesting possibility; namely a continuous brane distribution in an interval of time: $\tau_- \leq \tau \leq \tau_+$ with $\sigma(\tau) = 0$. In these circumstances, the effective Brane-action takes the following form:

$$\mathcal{S}_B = \int d\tau' |g_{00}| d\tau \delta(\tau - \tau') \Theta(\tau' - \tau_-) \Theta(\tau_+ - \tau') d^{d-1}x \sqrt{g_\perp} \left\{ e^{-2\phi} \left[\frac{\mathcal{R}}{2} + 2(\partial\phi)^2 \right] + P - V(\phi) \right\} + \mathcal{S}_- + \mathcal{S}_+. \quad (3.4)$$

Introducing the time interval $D \equiv (\tau_-, \tau_+)$, the Brane-action can be written as:

$$\mathcal{S}_B = \int_D d^d x \sqrt{-g} \left\{ e^{-2\phi} \left[\frac{\mathcal{R}}{2} + 2(\partial\phi)^2 \right] + P - V(\phi) \right\} + \mathcal{S}_- + \mathcal{S}_+, \quad (3.5)$$

where the localized brane terms at τ_\pm are

$$\mathcal{S}_\pm = - \int d\tau \delta(\tau - \tau_\pm) d^{d-1}x \sqrt{g_\perp} V_\pm(\phi). \quad (3.6)$$

As we will see later these terms turn out to be crucial for achieving the gluing between the Brane with the Momentum and/or the Winding regime(s).

The string effective action and the resulting cosmological solutions (defined for all times), can be obtained once we combine consistently the three regimes, namely:

$$\mathcal{S} = \mathcal{S}_W + \mathcal{S}_B + \mathcal{S}_M, \quad (3.7)$$

so that the thermal modulus σ is unrestricted in \mathcal{S} , even though σ is restricted in the individual effective actions \mathcal{S}_W , \mathcal{S}_M and \mathcal{S}_B .

3.1 Consistency conditions for $V(\phi)$ in the Brane regime

In the Brane-action, we have introduced an effective potential $V(\phi)$. We will see in Section 4 that this potential is sourced by the extra massless states, which give rise to non-trivial gradients and fluxes at $\sigma = 0$, *i.e.* when the temperature reaches its maximal value $T = T_c$. Thus, during the Brane regime, the temperature must remain constant. This implies that the structure of the effective potential is not arbitrary but strongly restricted by the constancy of the temperature T during the time interval $D = (\tau_-, \tau_+)$. We are interested in homogeneous and isotropic backgrounds

$$ds^2 = -N(\tau)^2 d\tau^2 + a(\tau)^2 d\Omega_k^2, \quad (3.8)$$

where Ω_k is a $(d-1)$ -dimensional Einstein space with curvature $k \leq 0$. The equations of motion for the laps function N and the scale factor a are ($H \equiv \dot{a}/a$) :

$$(N) : \quad \frac{1}{2}(d-1)(d-2) \left(H^2 + k \frac{N^2}{a^2} \right) = 2(d-1)H\dot{\phi} - 2\dot{\phi}^2 + e^{2\phi}N^2(\rho + V), \quad (3.9)$$

$$(a) : \quad (d-2) \left(\frac{\ddot{a}}{a} - H \frac{\dot{N}}{N} \right) + \frac{1}{2}(d-2)(d-3) \left(H^2 + k \frac{N^2}{a^2} \right) = 2\ddot{\phi} + 2(d-2)H\dot{\phi} - 2\dot{\phi}^2 - 2\frac{\dot{N}}{N}\dot{\phi} - e^{2\phi}N^2(P - V). \quad (3.10)$$

The dilaton equation is given by

$$\ddot{\phi} + (d-1)H\dot{\phi} - \dot{\phi}^2 - \frac{\dot{N}}{N}\dot{\phi} - \frac{d-1}{2} \left(\frac{\ddot{a}}{a} - H \frac{\dot{N}}{N} \right) - \frac{1}{4}(d-1)(d-2) \left(H^2 + k \frac{N^2}{a^2} \right) = \frac{1}{4}e^{2\phi}N^2 \frac{dV}{d\phi}. \quad (3.11)$$

Combining the above equations and using the thermodynamical identity $\rho = -P - \frac{\partial}{\partial|\sigma|}P$, we obtain the entropy conservation equation,

$$(\dot{\rho} + \dot{P}) + ((d-1)H + |\dot{\sigma}|)(\rho + P) = 0, \quad (3.12)$$

which implies that in order for the temperature to be constant, attaining its maximal value $T = T_c$ (equivalently $\sigma = 0$), not only are the pressure and the energy density constant, ($\rho = \rho_c$, $P = P_c$), but also the scale factor must be constant: $a = a_c$ with $H = 0$. Then, in the conformal gauge $N \equiv a$, where $a \equiv a_c$, the equations of motion yield:

$$(N) : \quad \frac{1}{2}(d-1)(d-2)k = -2\dot{\phi}^2 + e^{2\phi}a_c^2(\rho_c + V), \quad (3.13)$$

$$(a) : \quad \frac{1}{2}(d-2)(d-3)k = 2\ddot{\phi} - 2\dot{\phi}^2 - e^{2\phi}a_c^2(P_c - V), \quad (3.14)$$

$$(\phi) : \quad \ddot{\phi} - \dot{\phi}^2 - \frac{1}{4}(d-1)(d-2)k = \frac{1}{4}e^{2\phi}a_c^2 \frac{dV}{d\phi}. \quad (3.15)$$

Combining Eqs (a) and (ϕ), we obtain an equation constraining the structure of the effective potential $V(\phi)$:

$$\frac{dV}{d\phi} + 2V = 2P_c - 2(d-2)\frac{k}{a_c^2}e^{-2\phi}. \quad (3.16)$$

This can be integrated to find:

$$V(\phi) = C e^{-2\phi} + B - 2(d-2)\frac{k}{a_c^2}\phi e^{-2\phi} \quad \text{where} \quad B = P_c \quad (3.17)$$

and C is an arbitrary integration constant. Eqs (N) and (a) then become:

$$\frac{1}{2}(d-1)(d-2)k = -2\dot{\phi}^2 + a_c^2[(\rho_c + P_c)e^{2\phi} + C] - 2(d-2)k\phi, \quad (3.18)$$

$$\frac{1}{2}(d-2)(d-3)k = 2\ddot{\phi} - 2\dot{\phi}^2 + a_c^2C - 2(d-2)k\phi. \quad (3.19)$$

In the following, only Eq. (3.18) needs to be solve, since it is easily seen to imply Eq. (3.19). Restricting to the case $k = 0$, the structure of the effective potential has a very suggestive form. Namely, it is given in terms of a binomial of $e^{-2\phi}$. Its origin has a well defined interpretation in terms of internal graviphoton and matter gauge field fluxes. We will return to this point in Section 4.

3.2 Cosmological evolution in the Brane regime, for $k = 0$

In the σ -model frame, the temperature T and the scale factor a are fixed to their critical values during the Brane regime. The only non-trivial evolution is that of the dilaton field, which can be easily derived in terms of the flux parameters C and B appearing in the effective potential. Actually, there is only one free parameter, since B is fixed to the critical value of the pressure, $B = P_c$.

In what follows, we restrict to the $k = 0$ case. When $C \geq 0$, we set the earlier endpoint of the Brane regime at infinity, $\tau_- = -\infty$. This means that for $\tau \leq \tau_+$, the Universe is in the Brane regime with a flat σ -model metric and constant critical temperature. The evolution of the dilaton as a function of conformal time ($N \equiv a$) is found by solving Eq. (3.18). When $C = 0$, one finds

$$C = 0 : \quad e^{-\phi} = a_c \sqrt{\frac{\rho_c + P_c}{2}}(-\tau), \quad a = a_c, \quad T = T_c, \quad \forall \tau \leq \tau_+ < 0. \quad (3.20)$$

The upper bound of τ is chosen to be strictly negative, $\tau_+ < 0$, in order to maintain the validity of the perturbative approach in the Brane regime. The choice of τ_+ determines the string coupling at the transition towards the Momentum phase, $\phi_+ := \phi(\tau_+)$. For $C > 0$, the solution takes the form:

$$C > 0 : \quad e^{-\phi} = \sqrt{\frac{\rho_c + P_c}{C}} \sinh \left[a_c \sqrt{\frac{C}{2}}(-\tau) \right], \quad a = a_c, \quad T = T_c, \quad \forall \tau \leq \tau_+ < 0, \quad (3.21)$$

where the upper bound $\tau_+ < 0$ guaranties the validity of the perturbative approach as before. For $C < 0$, the evolution of the dilaton becomes

$$C < 0 : \quad e^{-\phi} = \sqrt{\frac{\rho_c + P_c}{-C}} \sin \left[a_c \sqrt{\frac{-C}{2}} (-\tau) \right], \quad a = a_c, \quad T = T_c, \quad \forall \tau_- \leq \tau \leq \tau_+ < 0. \quad (3.22)$$

In this case, in order to avoid a second non-perturbative regime, we need to impose a lower bound for τ which satisfies

$$\tau_{\min} := -\frac{\pi}{a_c} \sqrt{\frac{2}{-C}} < \tau_-. \quad (3.23)$$

Thus, the Brane regime lasts for a suitable finite time interval in this case. The choice of τ_- determines the string coupling when the Universe enters the Brane regime from the Winding phase, while τ_+ controls the dilaton at the exit of the Brane regime towards the Momentum phase, $\phi_{\pm} := \phi(\tau_{\pm})$.

Although the metric is flat in the σ -model frame, it acquires a non-trivial time-dependence in the Einstein frame defined for $d > 2$, due to the rescaling by a dilaton-dependent factor: $g_{\mu\nu}^E \equiv e^{-\frac{4\phi}{d-2}} g_{\mu\nu}$. Consequently, the laps function, scale factor and temperature in the Einstein frame are given by

$$N_E = N e^{-\frac{2\phi}{d-2}}, \quad a_E = a_c e^{-\frac{2\phi}{d-2}}, \quad T_E = T_c e^{\frac{2\phi}{d-2}}. \quad (3.24)$$

To determine them in the cosmological frame defined as

$$ds_E^2 = -dt^2 + a_E^2(t) d\vec{x}^2, \quad (3.25)$$

we rewrite Eq. (3.18) in the gauge $N \equiv e^{\frac{2\phi}{d-2}}$:

$$2\dot{\phi}^2 = e^{\frac{4\phi}{d-2}} [(\rho_c + P_c) e^{2\phi} + C]. \quad (3.26)$$

To interpret the above equation, we express it in terms of the scale factor in the Einstein frame. Defining $H_E \equiv \dot{a}_E/a_E$, the dilaton kinetic energy on the left hand side is proportional to H_E^2 , while on the right hand side, the terms with coefficients $(\rho_c + P_c)$ and C are proportional to $1/a^d$ and $1/a^2$, respectively. To be specific, one obtains

$$\frac{1}{2}(d-1)(d-2) \left(H_E^2 + \frac{k_{\text{eff}}}{a_E^2} \right) = \frac{r_{\text{eff}}}{a_E^d}, \quad (3.27)$$

where

$$k_{\text{eff}} = -C \frac{2a_c^2}{(d-2)^2} \quad \text{and} \quad r_{\text{eff}} = \frac{d-1}{d-2} (\rho_c + P_c) a_c^d. \quad (3.28)$$

This shows that effectively, the evolution of the Universe is identical to that of a spatially curved space filled with thermal radiation. In fact, denoting by n_{eff}^* the effective number of massless degrees of freedom associated to this radiation-like energy density,

$$\frac{r_{\text{eff}}}{a_c^d} := (d-1) \frac{n_{\text{eff}}^* \Sigma_d}{\beta_c^d}, \quad (3.29)$$

we find n_{eff}^* is larger than n^* ,

$$n_{\text{eff}}^* = \frac{d}{d-2} n^*. \quad (3.30)$$

When $C = 0$, the thermal energy ρ_c and the flux contribution $B = P_c$ yield a radiation-like evolution

$$C = 0 : \quad e^{-\frac{d\phi}{d-2}} = \left(\frac{T_c}{T_E} \right)^{\frac{d}{2}} = \left(\frac{a_E}{a_c} \right)^{\frac{d}{2}} = \frac{d}{d-2} \sqrt{\frac{\rho_c + P_c}{2}} (-t), \quad \forall t \leq t_+ < 0, \quad (3.31)$$

where the upper bound t_+ of the cosmological time is chosen negative, in order to avoid a non-perturbative regime and a formal Big Crunch at $t = 0$. To find the evolution when $C \neq 0$, we define $s = \text{sign}(C)$ and

$$v = \left(\frac{|C|}{\rho_c + P_c} \right)^{\frac{d}{2(d-2)}} \left(\frac{a_E}{a_c} \right)^{\frac{d}{2}}, \quad (3.32)$$

to rewrite the effective Friedmann equation (3.27) as

$$\frac{dv}{\sqrt{1 + s v^{\frac{2(d-2)}{d}}}} = \pm \Omega dt \quad \text{where} \quad \Omega = \frac{d}{d-2} \sqrt{\frac{\rho_c + P_c}{2}} \left(\frac{|C|}{\rho_c + P_c} \right)^{\frac{d}{2(d-2)}}. \quad (3.33)$$

Defining the primitive

$$f_d^s(v) = \int_0^v \frac{dz}{\sqrt{1 + s z^{\frac{2(d-2)}{d}}}} \equiv v {}_2F_1\left(\frac{1}{2}, \frac{d}{2(d-2)}; 1 + \frac{d}{2(d-2)}; -s v^{\frac{2(d-2)}{d}}\right), \quad (3.34)$$

we obtain

$$f_d^s(v) = -\Omega t, \quad \forall t < 0. \quad (3.35)$$

The function $f_d^+(v)$ for $v > 0$ is monotonically increasing and can be inverted. As a result, the cosmological evolution for $C > 0$ takes the form

$$C > 0 : \quad e^{-\frac{d\phi}{d-2}} = \left(\frac{T_c}{T_E} \right)^{\frac{d}{2}} = \left(\frac{a_E}{a_c} \right)^{\frac{d}{2}} = \left(\frac{\rho_c + P_c}{C} \right)^{\frac{d}{2(d-2)}} f_d^{+(-1)}(-\Omega t), \quad \forall t \leq t_+ < 0. \quad (3.36)$$

The choice of an upper bound $t_+ < 0$ for the cosmological time guaranties that the Universe passes into the Momentum phase and avoids a strong coupling regime. The function $f_d^-(v)$ is defined for $0 < v \leq 1$, where it is monotonically increasing and invertible. The time-evolution we find is

$$C < 0 : e^{-\frac{d\phi}{d-2}} = \left(\frac{T_c}{T_E}\right)^{\frac{d}{2}} = \left(\frac{a_E}{a_c}\right)^{\frac{d}{2}} = \left(\frac{\rho_c + P_c}{-C}\right)^{\frac{d}{2(d-2)}} f_d^{(-1)}(-\Omega t), \quad \forall t_- \leq t \leq t_+ < 0. \quad (3.37)$$

In order to avoid two non-perturbative regimes, time is restricted to a range $t \in (t_-, t_+)$. As before, t_+ must be strictly negative, while t_- satisfies

$$t_{\min} := -\frac{2}{\Omega} f_d^{(-1)}(1) \equiv -\frac{2\sqrt{\pi}}{\Omega} \frac{\Gamma(1 + \frac{d}{2(d-2)})}{\Gamma(1 + \frac{1}{d-2})} < t_-. \quad (3.38)$$

In fact, the formal solution for $C < 0$ and $t_{\min} < t < 0$ admits a symmetry, $t \rightarrow t_{\min} - t$. The latter maps the formal non-perturbative Big Crunch at $t = 0$ to a formal non-perturbative Big Bang at $t = t_{\min}$. Moreover, the scale factor bounces when it reaches its maximum at $t = t_{\min}/2$. The qualitative behavior of the above solutions for arbitrary C is independent of the dimension $d > 2$. For instance, in the physically interesting case $d = 4$, the analytic functions $f_d^{s(-1)}$ happen to be polynomials of degree two and the solutions take the explicit form

$$\forall C \in \mathbb{R} : e^{-2\phi} = \left(\frac{T_c}{T_E}\right)^2 = \left(\frac{a_E}{a_c}\right)^2 = \sqrt{2(\rho_c + P_c)} (-t) + \frac{C}{2} t^2. \quad (3.39)$$

The picture in Einstein frame can be summarized as follows:

- For $C = 0$, we get effectively a contracting “radiation-like” era ($H_E^2 \propto 1/a_E^d$), which starts at weak coupling. No singular behavior is encountered at large negative times, consistently with the choice $t_- = -\infty$. The Big Crunch and strong coupling regime at $t = 0$ is avoided since the endpoint of the Brane regime is located at $t_+ < 0$.
- For $C > 0$, we start with a weakly coupled contracting “curvature-like” dominated era ($k_{\text{eff}} < 0$ and $H_E^2 \sim 1/a_E^2$), followed by a radiation dominated era. Here also, the Big Crunch and strong coupling regime is avoided since $t_+ < 0$.
- For $C < 0$, the effective curvature is positive ($k_{\text{eff}} > 0$) and the mathematical solution starts formally with a Big Bang at infinite coupling, bounces and ends in a formal Big Crunch at infinite coupling. However, restricting time to the interval (t_-, t_+) guaranties

the Brane regime to be compatible with our perturbative approximation. Depending on the fact that $t_{\min}/2$ may be lower than t_- , in the interval (t_-, t_+) or larger than t_+ , the scale factor is either decreasing, bouncing or increasing.

- In the cases we are mostly interested, we have $C \geq 0$ and t_- can be consistently taken to $-\infty$. No singularity is encountered since $t_+ < 0$. We exit from the Brane regime at $t = t_+$, when the dilaton reaches a sufficiently large value ϕ_+ for the brane to decay into the conventional perturbative radiation plus moving dilaton era.

3.3 Cosmological evolution in the Momentum or Winding regimes, for $k = 0$

For $\tau > \tau_+$, the Brane potential $V(\phi)$ and the localized terms responsible for the gluing of the Brane and Momentum regimes are absent. The evolution of the scale factor a , the temperature T and the dilaton field ϕ are dictated by the Momentum effective action \mathcal{S}_M given in Eq. (3.1). In Ref. [15], the cosmological evolution in this regime was determined for $d \geq 2$. The scale factor and temperature in σ -model frame are expressed in terms of the positive thermal modulus σ ,

$$\frac{a}{a_c} = \frac{T_c}{T} = e^\sigma. \quad (3.40)$$

Under the approximation of Eq. (2.4), which implies the state equation for radiation $\rho \simeq (d-1)P$, the modulus σ and the dilaton field can be found analytically. For $d > 2$ and in conformal gauge ($N \equiv a$), they are given by²

$$\begin{aligned} \sigma(\tau) &= \frac{1}{d-2} \left[\eta_+ \ln \left(1 + \frac{\omega_+(\tau - \tau_+)}{\eta_+} \right) - \eta_- \ln \left(1 + \frac{\omega_+(\tau - \tau_+)}{\eta_-} \right) \right], \quad \forall \tau > \tau_+, \\ \phi(\tau) &= \phi_+ + \frac{\sqrt{d-1}}{2} \left[\ln \left(1 + \frac{\omega_+(\tau - \tau_+)}{\eta_+} \right) - \ln \left(1 + \frac{\omega_+(\tau - \tau_+)}{\eta_-} \right) \right], \end{aligned} \quad (3.41)$$

where $\eta_\pm = \sqrt{d-1} \pm 1$. The parameter ω_+ turns out to be related to the thermal energy density ρ , the scale factor and dilaton field at the instant τ_+ , when the Brane and Momentum regimes are glued:

$$\omega_+ = \frac{d-2}{\sqrt{2(d-1)}} a_c \sqrt{\rho_c} e^{\phi_+}. \quad (3.42)$$

²A solution for $d = 2$ also exists [14]. It can be recovered by taking the limit $d \rightarrow 2_+$ in Eq. (3.41) [15]. In the Hybrid model, Eq. (2.4) with $d = 2$ is exact.

In the Brane regime, the σ -model metric is static and the Ricci scalar vanishes. In the Momentum regime, the Ricci scalar is decreasing with time and its maximal value is reached at the brane exit,

$$\mathcal{R}_+ = 2\rho_c e^{2\phi_+}. \quad (3.43)$$

Thus, both higher derivative corrections and higher genus contributions remain small throughout the Momentum regime and can be consistently neglected, provided the critical value of the string coupling e^{ϕ_+} is taken sufficiently small. Far from the brane ($\omega_+(\tau - \tau_+) \gg 1$), the dilaton is asymptotically constant, the temperature drops and the scale factor tends to infinity.

When the Brane regime is characterized by a potential $V(\phi)$ with a parameter $C < 0$, it must last a finite amount of time (τ_-, τ_+) preceded by a Winding phase. For $\tau < \tau_-$, the thermal modulus is negative,

$$\frac{a}{a_c} = \frac{T_c}{T} = e^{-\sigma}. \quad (3.44)$$

It follows together with the dilaton field a time-evolution similar to that of the Momentum phase,

$$\begin{aligned} \sigma(\tau) &= \frac{-1}{d-2} \left[\eta_+ \ln \left(1 + \frac{\omega_-(\tau_- - \tau)}{\eta_+} \right) - \eta_- \ln \left(1 + \frac{\omega_-(\tau_- - \tau)}{\eta_-} \right) \right], \quad \forall \tau < \tau_-, \\ \phi(\tau) &= \phi_- + \frac{\sqrt{d-1}}{2} \left[\ln \left(1 + \frac{\omega_-(\tau_- - \tau)}{\eta_+} \right) - \ln \left(1 + \frac{\omega_-(\tau_- - \tau)}{\eta_-} \right) \right], \end{aligned} \quad (3.45)$$

where

$$\omega_- = \frac{d-2}{\sqrt{2(d-1)}} a_c \sqrt{\rho_c} e^{\phi_-}. \quad (3.46)$$

The Momentum regime and the eventual Winding regime evolutions can be described in cosmological frame defined in Eq. (3.25) by solving the associated Friedmann equation. The latter takes the form

$$\frac{1}{2}(d-1)(d-2) H_E^2 = \frac{r_{\pm}}{a_E^d} + \frac{c_{\pm}}{a_E^{2(d-1)}}, \quad (3.47)$$

where

$$r_{\pm} = \rho_c a_c^d \quad \text{and} \quad c_{\pm} = \frac{1}{d-2} \rho_c e^{-2\phi_{\pm}} a_c^{2(d-1)}. \quad (3.48)$$

In the right hand side, the first contribution arises from the thermal radiation, while the second is the kinetic energy density of the dilaton modulus. Clearly, the late time evolution of the Universe ($t \gg |t_+|$) and the eventual early time one ($t \ll t_-$) are radiation dominated,

while the contribution of the energy stored in the dilaton motion increases as we approach the transitions to the Brane regime, $(t - t_+ \rightarrow 0_+, t - t_- \rightarrow 0_-)$.

3.4 Gluing the Brane, Momentum and Winding regimes

Assuming $\tau_- = -\infty$, we first consider the string effective action which is valid in both the Brane and the Momentum regimes,

$$\mathcal{S}_{BM} = \int d^d x \sqrt{-g} \left\{ e^{-2\phi} \left[\frac{\mathcal{R}}{2} + 2(\partial\phi)^2 \right] + P - V(\phi)\Theta(\tau_+ - \tau) \right\} - \int d^d x \sqrt{g_\perp} e^{-2\phi} \kappa_+ \delta(\tau - \tau_+). \quad (3.49)$$

In this expression, we use the potential defined in Eq. (3.6), which is valid at the endpoint τ_+ of the brane regime: $V_+(\phi) = \kappa_+ e^{-2\phi}$. The latter is derived in Refs [14, 15] and reviewed in Section 4.1. On the contrary, the effective potential $V(\phi)$ in the Brane regime arises from the distribution of thin branes in the time-interval $-\infty < \tau < \tau_+$. The above action leads to the following equations of motion:

- For N :

$$\frac{1}{2}(d-1)(d-2) \left(H^2 + k \frac{N^2}{a^2} \right) = 2(d-1)H\dot{\phi} - 2\dot{\phi}^2 + e^{2\phi} N^2 (\rho + V\Theta(\tau_+ - \tau)). \quad (3.50)$$

- For a :

$$(d-2) \left(\frac{\ddot{a}}{a} - H \frac{\dot{N}}{N} \right) + \frac{1}{2}(d-2)(d-3) \left(H^2 + k \frac{N^2}{a^2} \right) = 2\ddot{\phi} + 2(d-2)H\dot{\phi} - 2\dot{\phi}^2 - 2\frac{\dot{N}}{N}\dot{\phi} - e^{2\phi} N^2 (P - V\Theta(\tau_+ - \tau)) + \kappa_+ N \delta(\tau - \tau_+). \quad (3.51)$$

- For ϕ :

$$\ddot{\phi} + (d-1)H\dot{\phi} - \dot{\phi}^2 - \frac{\dot{N}}{N}\dot{\phi} - \frac{d-1}{2} \left(\frac{\ddot{a}}{a} - H \frac{\dot{N}}{N} \right) - \frac{1}{4}(d-1)(d-2) \left(H^2 + k \frac{N^2}{a^2} \right) = \frac{1}{4} e^{2\phi} N^2 \frac{dV}{d\phi} \Theta(\tau_+ - \tau) - \frac{1}{2} \kappa_+ N \delta(\tau - \tau_+). \quad (3.52)$$

The above differential system implies that $\dot{\phi}$ is discontinuous across the endpoint τ_+ of the Brane, while \dot{a} is smooth. Since the scale factor remains constant during the Brane regime, $a \equiv a_c$ for $\tau < \tau_+$, it follows that $\dot{a}(\tau_+) = 0$ at the transition to the Momentum phase.

Consistently, this condition is fulfilled by the evolution displayed in Eqs (3.40) and (3.41). The brane tension κ_+ is related to the discontinuity of $\dot{\phi}$ at $\tau = \tau_+$,

$$\kappa_+ = 2 \frac{\dot{\phi}(\tau_+)_{\text{B}} - \dot{\phi}(\tau_+)_{\text{M}}}{N(\tau_+)}, \quad (3.53)$$

as follows from Eq. (3.52). Utilizing the Friedmann equation (3.50) just before and after the brane endpoint τ_+ , the fact that $\dot{a}(\tau_+) = 0$ and restricting to the case $k = 0$, we obtain for $C \geq 0$,

$$\dot{\phi}(\tau_+)_{\text{B}} = \frac{N(\tau_+)}{\sqrt{2}} \sqrt{V(\phi_+) + \rho_c} e^{\phi_+}, \quad \dot{\phi}(\tau_+)_{\text{M}} = -\frac{N(\tau_+)}{\sqrt{2}} \sqrt{\rho_c} e^{\phi_+}, \quad (3.54)$$

in terms of ρ_c and $V(\phi_+) = P_c + C e^{-2\phi_+}$, where ϕ_+ is the value of the dilaton field at τ_+ . Combining the above expressions, we obtain the brane tension κ_+ ,

$$C \geq 0: \quad \kappa_+ = \sqrt{2} \left(\sqrt{C + (\rho_c + P_c) e^{2\phi_+}} + \sqrt{\rho_c} e^{\phi_+} \right). \quad (3.55)$$

Thus, κ_+ is determined by the parameter C , modulo corrections in string coupling, $\kappa_+ = \sqrt{2C} + \mathcal{O}(e^{\phi_+})$. Note that the gluing of Winding and Brane regimes at some earlier instant τ_- would be incompatible with the positivity condition of a tension κ_- at the transition. This is the reason why we set $\tau_- = -\infty$ and consider a gluing at τ_+ only.

On the contrary, as we already stated in Section 3.2, the case $C < 0$ requires the Brane regime to be restricted to a finite range of time, in order to avoid two non-perturbative regimes. The relevant effective action is the combination of the Winding-, Brane- and Momentum-actions: \mathcal{S}_{WBM} . Reasoning as before, the brane tension κ_+ that triggers the Brane to Momentum phase transition is found to be

$$C < 0: \quad \kappa_+ = \sqrt{2} \left(\sqrt{\rho_c} e^{\phi_c} + \epsilon_+ \sqrt{(\rho_c + P_c) e^{2\phi_c} - |C|} \right), \quad (3.56)$$

where

$$\epsilon_+ = \text{sign}\left(\tau_+ - \frac{\tau_{\min}}{2}\right) \quad \text{and} \quad \tau_{\min} + \frac{1}{a_c} \sqrt{\frac{2}{-C}} \arcsin \left(\sqrt{\frac{P_c}{\rho_c + P_c}} \right) \leq \tau_+ < 0. \quad (3.57)$$

As compared to Eqs (3.22) and (3.23), the stronger constraint on the allowed choices of upper bound τ_+ of the Brane regime follows from the positivity constraint to be imposed on κ_+ . For $\tau < \tau_-$, we are in the Winding regime describing a contracting Universe with

increasing dilaton. At the transition from the Winding to the Brane regimes occurring at $\tau = \tau_-$, we have a tension

$$C < 0 : \quad \kappa_- = \sqrt{2} \left(\sqrt{\rho_-} e^{\phi_c} + \epsilon_- \sqrt{(\rho_c + P_c) e^{2\phi_-} - |C|} \right), \quad (3.58)$$

where ϕ_- is the dilaton field at τ_- ,

$$\epsilon_- = \text{sign}\left(\frac{\tau_{\min}}{2} - \tau_-\right) \quad \text{and} \quad \tau_{\min} \leq \tau_- < -\frac{1}{a_c} \sqrt{\frac{2}{-C}} \arcsin \left(\sqrt{\frac{P_c}{\rho_c + P_c}} \right). \quad (3.59)$$

Again, the positivity condition on κ_- reduces the upper bound of the allowed values of the instant τ_- of entry into the Brane regime.

3.5 String non-singular cosmologies

As we have shown in the previous Sections and in Refs [14,15], string theory provides the tools for realizing a gluing mechanism between different effective field theories valid in different cosmological regimes. This stringy gluing mechanism is possible due to the appearance of extra Euclidean massless states, localized at times when the temperature reaches its maximal critical value, which possess non-trivial thermal-momentum and thermal-winding charges. These states induce phase transitions from the Winding to the Brane regime and from the Brane to the Momentum regime, consistently with string perturbation theory. Following our previous considerations, we find some interesting classes of singularity-free string cosmological solutions:

- (i) $\{\mathcal{C}_{\text{String}}(\tau)\} \equiv \{\mathcal{W}(\tau < \tau_-)\} \oplus \{\mathcal{B}(\tau_- \leq \tau \leq \tau_+)\} \oplus \{\mathcal{M}(\tau_+ < \tau)\}$,
- (ii) $\{\tilde{\mathcal{C}}_{\text{String}}(\tau)\} \equiv \{\mathcal{B}(\tau \leq \tau_+)\} \oplus \{\mathcal{M}(\tau_+ < \tau)\}$.

The first class (i) generalizes the bouncing solutions of Refs [14,15], spreading the phase transition between the Winding and Momentum regimes within a finite lapse of time. During this interval, the σ -model temperature and scale factor are constant, attaining their critical values. This intermediate regime precisely corresponds to the Brane regime. In this phase, the effective potential $V(\phi) = B + C e^{-2\phi}$ requires $C < 0$, while B is fixed by the equations of motion to be equal to the critical value of the pressure, $B = P_c$.

The second class (ii) describes non-singular string cosmologies, where the Universe is in the Brane regime at very early times, $\tau \ll \tau_+$, with a flat σ -model metric, a constant

maximal temperature T_c and weak string interactions, $g_{\text{str}} \ll 1$. The Brane effective potential $V(\phi) = B + Ce^{-2\phi}$ must have $C \geq 0$, while $B = P_c$ as in case (i). During the evolution, the metric remains flat up to τ_+ , while the string coupling grows and reaches a critical value g_{str}^* at τ_+ , where the Universe exits from the Brane regime. At later times, $\tau > \tau_+$, the Universe is in a new phase in expansion, with radiation and decreasing string coupling due to the dilaton motion. Throughout the evolution, the string coupling remains smaller than g_{str}^* .

In the Einstein cosmological frame, the time-evolution in case (ii) gives rise to a new class of bouncing Universes connecting at $t = t_+$ two distinct phases. The initial contracting phase is characterized by a uniform time-distribution of spacelike branes and fluxes up to t_+ . When $C > 0$, the scale factor and temperature behave as in a negatively curved space with radiation. In the special $C = 0$ case, the effective curvature vanishes and the Brane regime is a radiation-like era. Actually, this effective behavior arises from the non-trivial motion of the dilaton field and the definitions of the scale factor and temperature in the Einstein frame, $a_E = a_c g_{\text{str}}^{-\frac{2}{d-2}}$, $T_E = T_c g_{\text{str}}^{\frac{2}{d-2}}$. In the second phase, $t > t_+$, the Universe is in expansion, with thermal radiation and a running dilaton converging to a constant.

In both cases (i) and (ii), the entropy is conserved during the evolution. The largeness of the entropy observed at late cosmological times implies that the size of the Universe during the Brane regime, where the temperature attains its maximal critical value, is already large. This fact guarantees the validity of the perturbative approach during the whole cosmological evolution, and manifests the connection of the so-called entropy and oldness problems of standard Big Bang cosmology.

4 Microscopic origin of the action in the Brane regime

In this Section, we investigate the microscopic origin of the effective action in the Brane regime, namely, the localized-in-time terms at the endpoints of the Brane (δ -functions proportional to κ_{\pm}), as well as the Brane effective potential ($V(\phi) = B + Ce^{-2\phi}$). We show that they follow naturally from the underlying description of the stringy system at the extended symmetry point, where the temperature reaches its maximal critical value T_c .

It is convenient to treat the microscopic action accordingly, when we are:

- (1) “Going in” and/or “going out” of the Brane, or

(2) “Being” on the Brane.

In case (1), the constraint $\delta(\sigma)$ is translated into localized constraints at the temporal endpoints of the brane: $\delta(\tau - \tau_-)$ and $\delta(\tau - \tau_+)$. In this case, the relevant microscopic action is described by a Euclidean action in $(d - 1)$ dimensions. In Refs [14, 15], this situation has been considered in great detail. The origin of a localized negative contribution to the pressure arises from non-trivial gradients in the spatial directions, $\langle \partial_{\hat{\mu}} \varphi^I \rangle \neq 0$, which the extra massless scalars at the endpoints of the brane can have, as we review in the next Section.

In case (2), the microscopic stringy system is described by an exact conformal theory at the extended symmetry point $\sigma = 0$, during the time-interval $D \equiv (\tau_-, \tau_+)$. Here, the constraint $\delta(\sigma)$ cannot be translated in time. The effective field theory in the lapse of time D is well described (in the Euclidean) by a $(d + 2)$ -dimensional supergravity theory, where the two extra dimensions are compactified on an $SU(2)_{k=2}/U(1)$ manifold.

As we will show explicitly, non-trivial gauge field fluxes during the interval of time D give rise to a d -dimensional effective potential of the form $V(\phi) = B + C e^{-2\phi}$, where $B, C \geq 0$. It is of the precise form to maintain in the Brane regime the constraint $\sigma = 0$. We will also comment on the possibility of generating strictly negative C -terms and refer to Ref. [28] for details on this issue.

4.1 The localized terms at the endpoints of the Brane regime

As we already stated, microscopically the localized terms at the endpoints of the brane arise due to the non-trivial gradients, $\langle \partial_{\hat{\mu}} \varphi^I \rangle \neq 0$, which the extra massless scalars present at $\sigma = 0$ acquire. These extra scalars give rise to a $(d - 1)$ -dimensional Euclidean action:

$$\mathcal{S}_{\pm} = - \int d\sigma d^{d-1}x \sqrt{g_{\perp}} e^{-2\phi} g^{\hat{\mu}\hat{\nu}} G_{IJ} \partial_{\hat{\mu}} \varphi^I \partial_{\hat{\nu}} \varphi^J \delta(\sigma), \quad (4.1)$$

where $\hat{\mu} = 1, \dots, d - 1$, $g_{\perp} = \det(g_{\hat{\mu}\hat{\nu}})$ and G_{IJ} is the φ -dependent metric in the field configuration space. The equations of motion of the scalars φ^I take the form:

$$2\partial_{\hat{\mu}}(e^{-2\phi} \sqrt{g_{\perp}} g^{\hat{\mu}\hat{\nu}} G_{IJ} \partial_{\hat{\nu}} \varphi^J) - e^{-2\phi} \sqrt{g_{\perp}} g^{\hat{\mu}\hat{\nu}} (\partial_I G_{KJ}) \partial_{\hat{\mu}} \varphi^K \partial_{\hat{\nu}} \varphi^J = 0. \quad (4.2)$$

They admit non-trivial solutions, which are consistent with the homogeneity and isotropy requirements and yield the Brane terms localized at the temporal endpoints τ_{\pm} [15]. The

relevant solutions are such that the induced metric,

$$h_{\hat{\mu}\hat{\nu}} \equiv G_{IJ} \partial_{\hat{\mu}} \varphi^I \partial_{\hat{\nu}} \varphi^J = \frac{\kappa_{\pm}}{d-1} g_{\hat{\mu}\hat{\nu}}, \quad (4.3)$$

is proportional to the spatial metric, where κ_+ and κ_- are positive constants. When this happens, the stress tensor of the scalars is consistent with the symmetries of the spatial metric, and therefore with homogeneity and isotropy. Moreover, the actions \mathcal{S}_+ and \mathcal{S}_- take the familiar form of the Nambu-Goto action for branes,

$$\begin{aligned} \mathcal{S}_{\pm} &= -\kappa_{\pm} \int d^d x e^{-2\phi} \sqrt{g_{\perp}} \delta(\tau - \tau_{\pm}) \\ &= -(d-1)^{\frac{d-1}{2}} \kappa_{\pm}^{\frac{3-d}{2}} \int d^d x e^{-2\phi} \sqrt{\det(G_{IJ} \partial_{\hat{\mu}} \varphi^I \partial_{\hat{\nu}} \varphi^J)} \delta(\tau - \tau_{\pm}). \end{aligned} \quad (4.4)$$

Thus, κ_{\pm} are interpreted as brane tensions. In the case $k = 0$, the isotropic embeddings $x^{\hat{\mu}} \rightarrow \varphi^I(x^{\hat{\nu}})$ are given by

$$\partial_{\hat{\mu}} \varphi^I = \sqrt{\frac{2\kappa_{\pm}}{d-1}} a_c \delta_{\hat{\mu}}^I, \quad (4.5)$$

so that both $g_{\hat{\mu}\hat{\nu}}$ and $G_{\hat{\mu}\hat{\nu}}$ are flat at $\tau = \tau_{\pm}$. As has been explained in Ref. [15], the realization of this isotropic embedding imposes a constraint on the dimensionality of space-time, namely $d \leq 6$.

4.2 The Brane effective potential V from field strength fluxes

At the extended symmetry point (where the Brane is located), the $U(1)$ Euclidean time circle is extended to an $SU(2)_{k=2}$ manifold, so that the target space is naturally $(d+2)$ -dimensional. Thus, instead of a “naive” field theory in d space-time dimensions, the Brane effective action during the time-interval $D \equiv (\tau_-, \tau_+)$ can be written as

$$\mathcal{S}_D = - \int d^{d+2} x \sqrt{g} \left\{ \mathcal{A}(\tilde{\phi}, \Phi^{\alpha}) (\partial_M \Phi^{\beta})^2 + \mathcal{B}(\tilde{\phi}, \Phi^{\alpha}) (F_{MN})_L^2 + \mathcal{C}(\tilde{\phi}, \Phi^{\alpha}) (F_{MN})_R^2 + \cdots \right\}, \quad (4.6)$$

where $\tilde{\phi}$ is the dilaton in $(d+2)$ -dimensions, the Φ^{α} 's are the internal massless scalars, $(F_{MN})_L$ is the field strength of the graviphotons, $(F_{MN})_R$ is the field strength of the matter gauge bosons. The ellipses denote the contributions of the metric, antisymmetric tensor and fermionic fields of the effective *gauged supergravity theory* [29] in $(d+2)$ dimensions. In the presence of non-trivial fluxes arising either from the gradients of the scalars or from the

gauge field strengths, an effective potential of the following form is generated for the dilaton in $(d + 2)$ dimensions,

$$V(\phi) = \tilde{\mathcal{A}}(\tilde{\phi}, \Phi^\alpha) + \tilde{\mathcal{B}}(\tilde{\phi}, \Phi^\alpha) + \tilde{\mathcal{C}}(\tilde{\phi}, \Phi^\alpha). \quad (4.7)$$

This form will persist when the theory is reduced to d dimensions, since the two extra compact dimensions of the $SU(2)_{k=2}/U(1)$ manifold have a finite volume of order 1 in string units. In order to derive the dilaton dependence of each individual term, we will utilize two facts:

- In the σ -model frame, the dilaton dependencies of the terms $\tilde{\mathcal{A}}$, $\tilde{\mathcal{B}}$ and $\tilde{\mathcal{C}}$ is independent of the dimensionality of space-time.

- The dilaton dependence of the gauge kinetic terms is dictated by the structure of the $\mathcal{N}_4 = 4$ supergravity theory, (gauged or not), in $d + 2 = 4$ space-time dimensions [29].

Combining these two properties, we will derive unambiguously how the three individual flux terms $\tilde{\mathcal{A}}$, $\tilde{\mathcal{B}}$ and $\tilde{\mathcal{C}}$ depend on the d -dimensional dilaton field ϕ ,

$$\tilde{\mathcal{A}} = A(\Phi^\alpha)e^{-2\phi}, \quad \tilde{\mathcal{B}} = B(\Phi^\alpha), \quad \tilde{\mathcal{C}} = \tilde{C}(\Phi^\alpha)e^{-2\phi}. \quad (4.8)$$

Let us discuss in more detail the origin of the action (4.6). The starting point is the type II superstring, with $n_c = 10 - d$ internal directions compactified on a torus T^{n_c} . The coupling (à la Scherck-Shwarz [30]) of at least one of the compact dimensions to right-moving R-symmetry charges breaks the initial $\mathcal{N}_4=(4,4)$ supersymmetry to a left-moving $\mathcal{N}_4=(4,0)$ supersymmetry. Thus, at least one of the n_c directions, say x^9 as in the model of Ref. [14], is compactified at the fermionic point and implies an $U(1)_{9R} \rightarrow [SU(2)_{9R}]_{k=2}$ extended symmetry. The temperature coupling to the left-moving sector is introduced in the compact Euclidean direction x^0 . At the fermionic point associated to the Brane regime, the $U(1)_{0L}$ symmetry of the temporal circle is extended to $[SU(2)_{0L}]_{k=2}$. Reorganizing the symmetries as,

$$\begin{aligned} \left(SU(2)_{0L} \otimes U(1)_{0R} \right) \times \left(U(1)_{9L} \otimes SU(2)_{9R} \right) = \\ \left(SU(2)_{0L} \otimes \left[U(1)_{0R} \times \frac{SU(2)_{9R}}{U(1)_{9R}} \right] \right) \times \left(U(1)_{9L} \otimes U(1)_{9R} \right), \end{aligned} \quad (4.9)$$

the temporal cycle S^1 is extended to a 3-dimensional sphere S^3 . This justifies why we considered the $\mathcal{N}_4 = (4, 0)$ supergravity theory in $(d + 2)$ dimensions in Eq. (4.6). The

introduction of the temperature coupling in the left-moving sector can be interpreted (in the Euclidean) as a gauging of the $\mathcal{N}_4 = (4, 0)$ supergravity in one less non-compact spatial dimension, but with 2 extra compact directions arising when the temperature is critical.

On the contrary, “outside the Brane” ($T < T_c$), the geometrical interpretation in $(d + 2)$ dimensions is not valid anymore. This statement is very crucial in what follows, since the spatial derivatives appearing in the gradients of the scalars and gauge kinetic terms in Eq. (4.6) will be taken in these two extra directions. As a result, the scalar and gauge fluxes considered above do not exist outside the Brane regime.

We are now in a position to determine the induced functional forms of the individual flux terms in Eq. (4.8). The easiest one is that of the scalar fields. The definition of the dilaton $\tilde{\phi}$ in $(d + 2)$ dimensions absorbs the volume factor $\sqrt{\det g_{ij}}$ of the n_c internal compact dimensions. Moreover, there is no dependence in the two extra directions (z, \bar{z}) , whose volume is finite and of order 1. Combining these two facts, we find in terms of the d -dimensional dilaton field that $\tilde{\mathcal{A}} = A e^{-2\tilde{\phi}}$, where A is a constant which absorbs the volume factor $\sqrt{\det g_{z\bar{z}}}$.

We now come to the \mathcal{B} and \mathcal{C} terms. Both are generated by some “gauge” fluxes in the spatial directions $(M, N) = (z, \bar{z})$. Restricting for the moment to the $d + 2 = 4$ dimensional case and according to the “gauged” $\mathcal{N}_4 = 4$ supergravity structure [29], we conclude that there are two possible dilaton dressings multiplying the gauge kinetic terms (in the Einstein-frame). This is due to the fact that the gauge bosons in the supergravity multiplet (left-moving graviphotons) and the gauge bosons in the matter supermultiplets transform non-trivially under the $SU(1, 1)$ duality symmetry of the dilaton-axion field. The latter parametrizes an $[SU(1, 1)/U(1)]_S$ manifold, where $S = e^{-2\tilde{\phi}} + i\chi$. (The axion field χ is the dual of the two-index antisymmetric tensor B_{MN}). The $SU(1, 1)$ symmetry implies a different coupling of S to the graviphotons and matter gauge fields:

$$\mathcal{B} (F_{MN}^2)_{\text{graviphotons}} + \mathcal{C} e^{-2\tilde{\phi}} (F_{MN}^2)_{\text{matter-gauge-fields}}. \quad (4.10)$$

In four dimensions, this expression takes an identical form, when written in the σ -model frame (due to fact that $\sqrt{\det g_{MN}} g^{PP'} g^{QQ'}$ is invariant under a conformal rescaling of the metric). The above constraints of $\mathcal{N}_4 = 4$ gauge supergravity in $d + 2 = 4$ dimensions turn out to be very efficient. Indeed, as announced before, the dilaton dependencies of the \mathcal{B} and \mathcal{C} terms in the σ -model frame remain the same in any dimension. Furthermore, assuming

non-trivial fluxes in the $(M, N) = (z, \bar{z})$ extra dimensions, and absorbing the volume factor of the n_c compact directions $\sqrt{\det g_{ij}}$ in the definition of the d -dimensional dilaton ϕ , we find that

$$\tilde{\mathcal{C}}(\tilde{\phi}, \Phi^\alpha) = \tilde{C} e^{-2\phi}, \quad \tilde{\mathcal{B}}(\tilde{\phi}, \Phi^\alpha) = B(\mathcal{U}), \quad (4.11)$$

where \tilde{C} is a constant, independent of the internal moduli fields Φ^α . As was the case for the $\tilde{\mathcal{A}}$ -term, there is no dependence in the (z, \bar{z}) extra dimensions, whose volume factor $\sqrt{\det g_{z\bar{z}}}$ is of order 1 and absorbed in the definitions of \tilde{C} and B . Since the $\tilde{\mathcal{B}}$ -term has no dilaton dependence, the Φ^α -dependent internal volume cannot be absorbed in the definition of the d -dimensional dilaton. Thus, although this term is independent of the dilaton, it does depend on the volume \mathcal{U} of the internal torus T^{n_c} .

We conclude, that the Brane effective potential for the dilaton ϕ in d dimensions, which is induced by the fluxes, takes the form

$$V(\phi) = B(\mathcal{U}) + C e^{-2\phi}, \quad (4.12)$$

where $C = A + \tilde{C}$ is a constant and \mathcal{U} is the volume of the internal $(10 - d)$ -dimensional space. According to Section 3.1, the dynamical requirement of being on the brane fixes the volume \mathcal{U} such that $B(\mathcal{U}) = P_c$.

Notice that both the B and C terms are positive, once their microscopic origin is due to the above considered fluxes. This seems to exclude the $C < 0$ case studied in Section 3. A natural question arising at this point, is the possibility of generating an effective $C < 0$ from different microscopic considerations. The answer to this question is affirmative. Indeed, such a negative term can arise, when one considers an underlying conformal field theory with a central charge deficit at genus-0. This can be easily realized in four spacetime dimensions, when the 3-dimensional space is S^3 rather than flat. The effective C in this case is proportional to the central charge deficit, $C \propto \delta\hat{c} = -\frac{4}{(k+2)}$, where k is the level of $SU(2)_k \sim S^3$. We plan to analyze this possibility elsewhere [28].

5 Conclusions

In this work, we investigated cosmological consequences of the recently discovered stringy gluing mechanism between different string effective field theories. This mechanism is induced

by the appearance of extra massless string states possessing non-trivial thermal-momentum and thermal-winding charges, and localized at cosmological times when the temperature reaches its maximal critical value.

The effective field theory regime with constant σ -model frame temperature equal to its maximal value ($T = T_c$) admits a natural “brane interpretation”, with tensions κ_{\pm} given by the non-trivial gradients $\langle \partial_{\mu} \varphi^I \rangle \neq 0$, which the extra massless scalars acquire at the endpoints of the Brane regime. Non-trivial gauge fluxes on the Brane give rise to an effective dilaton potential, $V(\phi) = B(\mathcal{U}) + C e^{-2\phi}$, which turns out to be compatible with the requirement of “being on” the Brane, with constant temperature equal to its maximal value during the time interval $\tau_- < \tau < \tau_+$. The B -term is proportional to the volume of the internal space, and fixed to be equal to the critical pressure at T_c , $B(\mathcal{U}) = P_c$, by the dilaton and gravitational field equations of motion. The coefficient C turns out to be constant and behaves like an effective central charge “deficit/benefit” in the underlying two-dimensional worldsheet superconformal theory.

When $C \geq 0$, the earliest endpoint of the brane is forced to be at $\tau_- = -\infty$. This fact gives rise to a new class of non-singular string cosmologies: $\{\mathcal{C}_{String}(\tau)\} \equiv \{\mathcal{B}(\tau \leq \tau_+)\} \oplus \{\mathcal{M}(\tau_+ < \tau)\}$, where the Universe at very early times stays in the Brane regime, corresponding to an exact worldsheet conformal field theory. It is characterized by a constant maximal σ -model temperature T_c , flat σ -model metric and by super weak string interactions, $g_{str} \ll 1$. During the evolution, the temperature and the metric remain stationary up to τ_+ , while the string coupling grows and reaches a critical value g_{str}^* at τ_+ . At later times, $\tau > \tau_+$, the Universe enters in the Momentum phase, $\{\mathcal{M}(\tau > \tau_+)\}$, which is described by a radiation dominated expansion and decreasing string coupling converging to a constant.

In the Einstein cosmological frame, the time-evolutions of the $\{\mathcal{C}_{String}(\tau)\} \equiv \{\mathcal{B}(\tau \leq \tau_+)\} \oplus \{\mathcal{M}(\tau_+ < \tau)\}$ cosmologies, give rise to a new class of bouncing Universes connecting at $t = t_+$ two distinct phases. The initial one is contracting and characterized by a scale factor and temperature evolving as in a negatively curved space filled with radiation. At later times, the Universe enters in an expanding thermal phase with a decreasing dilaton, which is asymptotic to a constant.

When $C < 0$, the Brane regime has two endpoints, τ_{\pm} . In this case, the Universe has three characteristic phases: $\{\mathcal{C}_{String}(\tau)\} \equiv \{\mathcal{W}(\tau < \tau_-)\} \oplus \{\mathcal{B}(\tau_- \leq \tau \leq \tau_+)\} \oplus \{\mathcal{M}(\tau_+ < \tau)\}$.

Namely, for $\tau < \tau_-$, the Universe is in a contracting Winding regime, up to the time τ_- when the temperature and the string coupling reach their maximal critical values, T_c and g_{str}^* . At τ_- , the Universe enters in the Brane regime, which lasts for a finite amount of time, having constant temperature $T = T_c$. The string coupling decreases reaching a minimal value $g_{\text{str}}^{\text{min}}$ at $\tau_{\text{min}}/2$, and then grows again reaching its maximal value at τ_+ , the second endpoint of the Brane. At τ_+ , the Universe enters in the Momentum expanding regime, with decreasing temperature and string coupling, the latter being asymptotic to a constant. The $C < 0$ solutions with an intermediate Brane regime ($\tau_- < \tau < \tau_+$) extend the non-singular bouncing cosmologies of Refs [14, 15], where the Brane regime collapses to a single instant in time.

The cosmological solutions described in this work remain perturbative throughout the evolution, provided that the critical value of the string coupling at the endpoints of the brane is sufficiently small. An important property is that the entropy is conserved during the whole evolution, with the critical value of the scale factor at the endpoints of the Brane regime being determined by it and the maximal critical temperature. This class of bouncing cosmologies as well as the those of Refs [14, 15] are the first higher dimensional examples, where both the Hagedorn instability as well as the classical Big Bang singularity are successfully resolved, remaining in a perturbative regime throughout the evolution.

We have presented spatially flat solutions ($k = 0$). However, there are other interesting possibilities with $k \neq 0$. In particular, when the stringy gluing mechanism is applied to the $k > 0$ case, it is possible to realize a cyclic closed cosmology, where the apparent Big Bang and Big Crunch singularities are resolved by the appearance of a periodic array of branes. This work is currently under progress [28]. It would be interesting to apply the stringy gluing mechanism in other physically interesting problems, where paradoxes related to spacelike singularities appear, such as in the interior of black hole geometries.

Having at our disposal exact cosmological solutions, we can explicitly calculate the spectrum of fluctuations at early times, say at the time locations of the branes, determine their propagation at later cosmological times and compare them to the current and future observational data. This is possible since we have analytical control on the theory describing the brane. This work is currently under progress [31].

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References

- [1] R. Penrose, “Structure of space-time,” in Battelle Rencontres, 1967, Lectures in Mathematics and Physics, edited by C. M. DeWitt and J. A. Wheeler, pp. 121-235, Benjamin, New York, 1968.
S. W. Hawking and G. F. R. Ellis, “The large scale structure of space-time,” Cambridge University Press, Cambridge, 1973.
S. Hawking and G. F. R. Ellis, “Singularities in homogeneous world models,” Phys. Lett. **17** (1965) 246.
- [2] R. H. Brandenberger and C. Vafa, “Superstrings in the early universe,” Nucl. Phys. B **316** (1989) 391.
A. Tseytlin and C. Vafa, “Elements of string cosmology,” Nucl. Phys. B **372** (1992) 443 [arXiv:hep-th/9109048].
- [3] G. Veneziano, “Scale factor duality for classical and quantum strings,” Phys. Lett. B **265** (1991) 287.
M. Gasperini and G. Veneziano, “Pre-big bang in string cosmology,” Astropart. Phys. **1** (1993) 317 [arXiv:hep-th/9211021].
M. Gasperini, M. Maggiore and G. Veneziano, “Towards a nonsingular pre-big bang cosmology,” Nucl. Phys. B **494** (1997) 315 [arXiv:hep-th/9611039].

- R. Brustein, M. Gasperini and G. Veneziano, “Duality in cosmological perturbation theory,” *Phys. Lett. B* **431** (1998) 277 [arXiv:hep-th/9803018].
- M. Gasperini and G. Veneziano, “The pre-big bang scenario in string cosmology,” *Phys. Rept.* **373** (2003) 1 [arXiv:hep-th/0207130].
- [4] M. Gasperini, M. Giovannini and G. Veneziano, “Perturbations in a nonsingular bouncing universe,” *Phys. Lett. B* **569** (2003) 113 [arXiv:hep-th/0306113].
- M. Gasperini, M. Giovannini and G. Veneziano, “Cosmological perturbations across a curvature bounce,” *Nucl. Phys. B* **694** (2004) 206 [hep-th/0401112].
- M. Giovannini, “Vector fluctuations from multidimensional curvature bounces,” *Phys. Rev. D* **70** (2004) 103509 [hep-th/0407124].
- M. Giovannini, “Heating up the cold bounce,” *Class. Quant. Grav.* **21** (2004) 4209 [hep-th/0406098].
- M. Giovannini, “Dynamical back-reaction of relic gravitons,” *Phys. Rev. D* **73** (2006) 083505 [hep-th/0601097].
- [5] E. Kiritsis and C. Kounnas, “Dynamical topology change, compactification and waves in a stringy early universe,” [arXiv:hep-th/9407005].
- E. Kiritsis and C. Kounnas, “Dynamical topology change in string theory,” *Phys. Lett. B* **331** (1994) 51 [arXiv:hep-th/9404092].
- E. Kiritsis and C. Kounnas, “Dynamical topology change, compactification and waves in string cosmology,” *Nucl. Phys. Proc. Suppl.* **41** (1995) 311 [arXiv:gr-qc/9701005].
- E. Kiritsis and C. Kounnas, “String gravity and cosmology: Some new ideas,” arXiv:gr-qc/9509017.
- [6] J. J. Atick and E. Witten, “The Hagedorn transition and the number of degrees of freedom of string theory,” *Nucl. Phys. B* **310** (1988) 291.
- [7] C. Kounnas and B. Rostand, “Coordinate dependent compactifications and discrete symmetries,” *Nucl. Phys. B* **341** (1990) 641.
- [8] I. Antoniadis and C. Kounnas, “Superstring phase transition at high temperature,” *Phys. Lett. B* **261** (1991) 369.

- I. Antoniadis, J. P. Derendinger and C. Kounnas, “Nonperturbative temperature instabilities in $\mathcal{N} = 4$ strings,” Nucl. Phys. B **551** (1999) 41 [arXiv:hep-th/9902032].
- C. Kounnas, “Universal thermal instabilities and the high-temperature phase of the $\mathcal{N} = 4$ superstrings,” arXiv:hep-th/9902072.
- [9] J. L. F. Barbon and E. Rabinovici, “Touring the Hagedorn ridge,” arXiv:hep-th/0407236.
- J. L. F. Barbon, E. Rabinovici, “Closed string tachyons and the Hagedorn transition in AdS space,” JHEP **0203** (2002) 057. [hep-th/0112173].
- [10] J. L. Davis, F. Larsen and N. Seiberg, “Heterotic strings in two dimensions and new stringy phase transitions,” JHEP **0508** (2005) 035 [arXiv:hep-th/0505081].
- N. Seiberg, “Long strings, anomaly cancellation, phase transitions, T-duality and locality in the 2d heterotic string,” JHEP **0601** (2006) 057 [arXiv:hep-th/0511220].
- J. L. Davis, “The moduli space and phase structure of heterotic strings in two dimensions,” Phys. Rev. D **74** (2006) 026004 [arXiv:hep-th/0511298].
- [11] S. Chaudhuri, “Finite temperature bosonic closed strings: Thermal duality and the Kosterlitz-Thouless transition,” Phys. Rev. D **65** (2002) 066008 [arXiv:hep-th/0105110].
- [12] K. R. Dienes and M. Lennek, “Adventures in thermal duality (I): Extracting closed form solutions for finite temperature effective potentials in string theory,” Phys. Rev. D **70** (2004) 126005 [arXiv:hep-th/0312216].
- K. R. Dienes and M. Lennek, “Adventures in thermal duality (II): Towards a duality covariant string thermodynamics,” Phys. Rev. D **70** (2004) 126006 [arXiv:hep-th/0312217].
- [13] N. Matsuo, “Superstring thermodynamics and its application to cosmology,” Z. Phys. C **36** (1987) 289.
- J. Kripfganz and H. Perl, “Cosmological impact of winding strings,” Class. Quant. Grav. **5** (1988) 453.
- M. J. Bowick and S. B. Giddings, “High temperature strings,” Nucl. Phys. B **325** (1989) 631.

- R. Easther, B. R. Greene, M. G. Jackson and D. N. Kabat, “String windings in the early universe,” JCAP **0502** (2005) 009 [arXiv:hep-th/0409121].
- J. E. Lidsey, D. Wands and E. J. Copeland, “Superstring cosmology,” Phys. Rept. **337** (2000) 343 [arXiv:hep-th/9909061].
- T. Battefeld and S. Watson, “String gas cosmology,” Rev. Mod. Phys. **78** (2006) 435 [arXiv:hep-th/0510022].
- N. Kaloper, L. Kofman, A. D. Linde and V. Mukhanov, “On the new string theory inspired mechanism of generation of cosmological perturbations,” JCAP **0610** (2006) 006 [arXiv:hep-th/0608200].
- N. Kaloper and S. Watson, “Geometric precipices in string cosmology,” Phys. Rev. D **77** (2008) 066002 [arXiv:0712.1820 [hep-th]].
- R. H. Brandenberger, “String gas cosmology,” arXiv:0808.0746 [hep-th].
- B. Greene, D. Kabat and S. Marnerides, “Bouncing and cyclic string gas cosmologies,” Phys. Rev. D **80** (2009) 063526 [arXiv:0809.1704 [hep-th]].
- [14] I. Florakis, C. Kounnas, H. Partouche and N. Toumbas, “Non-singular string cosmology in a 2d Hybrid model,” Nucl. Phys. **B844** (2011) 89-114. [arXiv:1008.5129 [hep-th]].
- [15] C. Kounnas, H. Partouche, N. Toumbas, “Thermal duality and non-singular cosmology in d -dimensional superstrings,” Nucl. Phys. **B855** (2012) 280-307. [arXiv:1106.0946 [hep-th]].
- [16] C. Angelantonj, C. Kounnas, H. Partouche and N. Toumbas, “Resolution of Hagedorn singularity in superstrings with gravito-magnetic fluxes,” Nucl. Phys. B **809** (2009) 291 [arXiv:0808.1357 [hep-th]].
- [17] C. Kounnas, “Massive boson-fermion degeneracy and the early structure of the universe,” Fortsch. Phys. **56** (2008) 1143 [arXiv:0808.1340 [hep-th]].
- I. Florakis and C. Kounnas, “Orbifold symmetry reductions of massive boson-fermion degeneracy,” Nucl. Phys. B **820** (2009) 237 [arXiv:0901.3055 [hep-th]].
- [18] I. Florakis, C. Kounnas and N. Toumbas, “Marginal deformations of vacua with massive boson-fermion degeneracy symmetry,” Nucl. Phys. B **834** (2010) 273 [arXiv:1002.2427 [hep-th]].

- [19] D. Kutasov and N. Seiberg, “Number of degrees of freedom, density of states and tachyons in string theory and CFT,” Nucl. Phys. B **358** (1991) 600.
- [20] K. Dienes, “Modular invariance, finiteness, and misaligned supersymmetry: New constraints on the numbers of physical string states,” Nucl. Phys. B **429** (1994) 533 [arXiv:hep-th/9402006].
- [21] E. Witten, “Phases of $\mathcal{N} = 2$ theories in two-dimensions,” Nucl. Phys. **B403** (1993) 159-222. [hep-th/9301042].
- [22] See for instance and references therein:
 J. H. Schwarz, “Lectures on superstring and M theory dualities: Given at ICTP Spring School and at TASI Summer School,” Nucl. Phys. Proc. Suppl. **55B** (1997) 1-32. [hep-th/9607201].
 J. Polchinski, “String theory. Vol. 2: Superstring theory and beyond,” Cambridge, UK: Univ. Pr. (1998) 531 p.
- [23] P. H. Ginsparg and C. Vafa, “Toroidal compactification of nonsupersymmetric heterotic strings,” Nucl. Phys. B **289** (1987) 414.
 V. P. Nair, A. D. Shapere, A. Strominger and F. Wilczek, “Compactification of the twisted heterotic string,” Nucl. Phys. B **287** (1987) 402.
 S. P. Patil and R. Brandenberger, “Radion stabilization by stringy effects in general relativity,” Phys. Rev. D **71** (2005) 103522 [arXiv:hep-th/0401037].
- [24] F. Bourliot, J. Estes, C. Kounnas and H. Partouche, “Cosmological phases of the string thermal effective potential,” Nucl. Phys. B **830** (2010) 330 [arXiv:0908.1881 [hep-th]].
 J. Estes, C. Kounnas and H. Partouche, “Superstring cosmology for $\mathcal{N}_4 = 1 \rightarrow 0$ superstring vacua,” Fortsch. Phys. **59** (2011) 861-895. [arXiv:1003.0471 [hep-th]].
 J. Estes, L. Liu and H. Partouche, “Massless D-strings and moduli stabilization in type I cosmology,” JHEP **1106** (2011) 060 [arXiv:1102.5001 [hep-th]].
- [25] T. Catelin-Jullien, C. Kounnas, H. Partouche and N. Toumbas, “Thermal/quantum effects and induced superstring cosmologies,” Nucl. Phys. B **797** (2008) 137 [arXiv:0710.3895 [hep-th]].

- T. Catelin-Jullien, C. Kounnas, H. Partouche and N. Toumbas, “Induced superstring cosmologies and moduli stabilization,” Nucl. Phys. B **820**, 290 (2009) [arXiv:0901.0259 [hep-th]].
- F. Bourliot, C. Kounnas and H. Partouche, “Attraction to a radiation-like era in early superstring cosmologies,” Nucl. Phys. B **816** (2009) 227 [arXiv:0902.1892 [hep-th]].
- [26] C. M. Hull, “Doubled geometry and T-folds,” JHEP **0707** (2007) 080. [hep-th/0605149].
- C. M. Hull, “Global aspects of T-duality, gauged sigma models and T-folds,” JHEP **0710** (2007) 057. [hep-th/0604178].
- [27] C. Kounnas, N. Toumbas, J. Troost, “A wave-function for stringy universes,” JHEP **0708** (2007) 018. [arXiv:0704.1996 [hep-th]].
- [28] C. Kounnas, H. Partouche and N. Toumbas, *Work in progress*.
- [29] M. de Roo, “Matter coupling in $\mathcal{N} = 4$ supergravity,” Nucl. Phys. **B255** (1985) 515.
- M. de Roo, “Gauged $\mathcal{N} = 4$ matter couplings,” Phys. Lett. **B156** (1985) 331.
- M. de Roo and P. Wagemans, “Gauge matter coupling in $\mathcal{N} = 4$ supergravity,” Nucl. Phys. **B262** (1985) 644.
- [30] J. Scherk and J. H. Schwarz, “Spontaneous breaking of supersymmetry through dimensional reduction,” Phys. Lett. B **82** (1979) 60.
- R. Rohm, “Spontaneous supersymmetry breaking in supersymmetric string theories,” Nucl. Phys. B **237** (1984) 553.
- C. Kounnas and M. Porrati, “Spontaneous supersymmetry breaking in string theory,” Nucl. Phys. B **310** (1988) 355.
- S. Ferrara, C. Kounnas and M. Porrati, “ $\mathcal{N} = 1$ superstrings with spontaneously broken symmetries,” Phys. Lett. B **206** (1988) 25.
- S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, “Effective superhiggs and Str M^2 from four-dimensional strings,” Phys. Lett. B **194** (1987) 366.
- S. Ferrara, C. Kounnas and M. Porrati, “Superstring solutions with spontaneously broken four-dimensional supersymmetry,” Nucl. Phys. B **304** (1988) 500.

- [31] R. Brandenberger, C. Kounnas, H. Partouche, S. Patil and N. Toumbas, “Fluctuations in non-singular bouncing cosmologies from Type II superstrings,” *Work in progress*.